Holographic correlator array with selectable shift-invariance

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Abstract
Optical holographic correlators can perform many correlations simultaneously. Because the output plane must be divided among the individual templates in the system, for many systems shift-invariance limits the number of correlation templates than can be stored in one correlator. When the system is completely shift-invariant, the correlation peak from one correlator can shift into an area that has been reserved for a different template; in this case, a shifted version of one object might be mistaken for a well-centered version of a different object. This paper describes a technique to control the shift-invariance of a correlator system by moving the holographic material away from the Fourier plane.

Keywords: optical correlator, volume holography

1 Introduction
Image recognition systems built from large numbers of correlation templates can be powerful tools for many applications, including navigation, human-computer interfaces, and security.\textsuperscript{1} The effectiveness and robustness of such systems improves if a larger number of correlation templates is used. However, system speed decreases as the number of templates increases if templates are sequentially entered into the correlator. Holographic correlators though, are capable of performing multiple correlations in parallel, displaying the results on the same detector at the output. The responses from different templates are distinguished by dividing the output plane into individual domains, one for each template. If an input image shifts too much, the corresponding correlation peak will move into a domain designated for another template, leading to a false identification. Limiting the shift invariance of the system prevents this error from occurring. However, some amount of shift-invariance is necessary for robust recognition. The trade-off between shift-invariance and the number of templates used is an important design parameter for pattern recognition systems. The less shift-invariance a system has, the smaller the individual domains can be and therefore more templates can be used in parallel.

The use of volume holograms is one method through which the shift invariance can be reduced\textsuperscript{2,3}. Bragg selectivity can cause the correlation peak to disappear for shifts of the input image. Bragg selectivity in the plane defined by the reference and signal beams is much greater than that for the out-of-plane direction, leading to a very asymmetric correlation domain. This is a problem since many applications require symmetric correlation domains. The relatively weak control over the size of the domain in the out-of-plane direction also decreases the number of templates that can be stored in this direction.

In this paper we present a new method\textsuperscript{4} for controlling the shift invariance by shifting the hologram away from the Fourier plane, into the Fresnel region.

2 Theory
In the optical correlator, when the input image shifts the plane wave components at the Fourier plane all experience the same phase shift. This property results in shift-invariance for thin holographic correlators stored in the Fourier plane. However, if the hologram is recorded away from the Fourier plane the phase shift is not uniform across all plane wave components. As a result, the various components of a shifted input image begin to add destructively and the correlation peak eventually disappears. The further the holographic material is from the Fourier plane, the bigger the phase difference between the various component plane waves and the more shift invariance is reduced.

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Figure 1 shows the basic correlator system with the holographic material shifted a distance \( z_c \) from the Fourier plane. A transparency \( f_1(x_1,y_1) \), illuminated by monochromatic light of wavelength \( \lambda_n \), produces the disturbance \( g(x,y,z) \) in the Fresnel zone given (within the paraxial approximation and assuming \( z_c \) is small relative to \( F \)) by

\[
g(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(x_1,y_1) e^{j\frac{k}{2F}(x_1^2 + y_1^2)} e^{j\frac{k}{z} (x_1 x + y_1 y)} \, dx_1 \, dy_1
\]

where \( k = \frac{2\pi}{\lambda_n} \), \( F \) is the focal length of the lens, and \( z \) is the distance from the Fourier plane. A plane wave reference incident at an angle \( \theta \) interacts with the signal beam to record a hologram given by

\[
\left| e^{j(kx'\sin\theta' + z'\cos\theta')} + g(x', y', z') \right|^2
\]

where \( \theta' \) is the angle of the reference beam inside the medium.

To perform a correlation the hologram is illuminated with a new input image, \( f_2(x_2,y_2) \). Taking into account the index of refraction and making some simplifying assumptions the expected response of the system has been derived to be

\[
A_d(\Delta x, \Delta y) = \int \int f_1(x_1,y_1) f_2(x_1 + \Delta x, y_1 + \Delta y) \times e^{j(z_c + \pi d x) (\bar{z} - L_c/2) \alpha} \sin \left( \frac{L_c \alpha}{2\pi} \right) dx_1 \, dy_1
\]

where

\[
\alpha = \frac{k}{2nF^2} (x_1^2 - (x_1 + \Delta x)^2 + y_1^2 - (y_1 + \Delta y)^2) + kn\cos\theta' - nk
\]

\[
\Delta x = -nF(\sin\theta' - k_{dx}/k)
\]

\[
\Delta y = -nFk_{dy}/k
\]
Equation 3 is the cross-correlation between $f_1$ and $f_2$ with a sinc term from the Bragg-selectivity and an exponential term involving the position of the hologram, $z_c$. The exponential and the sinc term act as window functions on the correlation, attenuating the signal for non-zero values of $\Delta x$ and $\Delta y$, i.e. for deviations from the center of the correlation domain. For volume holograms recorded at the Fourier plane, the exponential term becomes identically equal to one and only the sinc term acts to limit shift invariance. For thin holograms recorded away from the Fourier plane the sinc term becomes negligible and the exponential term becomes the limiting factor. The presence of this window function within the integral also acts to sharpen the correlation peaks by suppressing the sidelobes, since they occur at non-zero values of $\Delta x$ and $\Delta y$ even when the input is centered. Attempting to store correlators too closely together results in the sidelobes from one template interfering with the neighboring template, reducing both outputs to noise. Issues concerning shift invariance aside, the sidelobes of the correlations place an upper limit on how tightly the correlations can be packed in a conventional correlator.

3 Experiment

The experimental setup for the correlator is shown in Figure 2. The liquid crystal spatial light modulator has a resolution of 640 by 480 pixels and a 24μm pixel pitch. A DC block in the Fourier plane of the first lens edge enhances the image before correlation. The filter behind the second lens blocks the edges of the SLM, created by the edge-enhancement process of the DC-block. If not blocked, the SLM outline creates an undesirable constant DC offset to the strength of the correlation. This DC offset is present regardless of what image is presented on the SLM and makes discrimination more difficult. The holographic material (a 250μm thick LiNbO$_3$ crystal) is mounted on a motorized translation stage to enable computerized control of the location relative to the Fourier plane. The signal beam is coincident with, and the reference beam at a 25° angle to the recording material surface normal. A lens is placed along the path of the reference beam and in its back focal plane a CCD camera is used to capture the intensity and position of the correlation peak. The video signal from the CCD camera is digitized and analyzed by a computer. A test image of random white and black rectangles, shown in Figure 3, was displayed on a portion of the liquid crystal spatial light modulator.

A hologram of the test pattern centered on the SLM is recorded for each hologram displacement distance $z_c$. After recording the reference beam is turned off and the image on the SLM is correlated with the stored hologram. To test the shift invariance the input image is electronically shifted on the SLM. The image is first shifted horizontally (the in-plane direction) while centered vertically. For each horizontal location, the intensity of the correlation peak and its location on the CCD is measured. The image is then shifted vertically (the out-of-plane direction) while centered horizontally, and again the peak intensity and position are measured. The correlation measurements are taken under very weak illumination to both prevent saturation of the CCD and erasure of the hologram.
Figure 3 – Randomly generated image 120 by 120 pixels in size

Figure 4 shows typical curves of peak intensity versus image location for both horizontal and vertical displacements. The shift-selectivity is measured as the width of the curves when they attain half of their maximum values. Plots of the shift-selectivity for both the in-plane and out-of-plane directions together with the theoretical predictions are shown in Figure 5 as func-

Figure 4 – Correlation strength versus image displacement for both in-plane (left) and out-of-plane (right) shifts

Figure 5 – In-plane (left) and out-of-plane (right) shift-selectivity with the material displaced from the Fourier plane
tions of the recording material location relative to the Fourier plane. The correlation integral from section 2 was computed with a Monte Carlo technique with experimental values for beam angle (25°), material thickness (250µm), and index of refraction (2.24).

The experiment agrees well with the theoretical calculations over a large range of material displacements. Theory and experiment deviate most for out-of-plane shifts close to and at the Fourier plane, where the predicted value of the shift-invariance shoots up to 25 degrees. The figure shown does not contain the full vertical range of the theoretical curve so that the details of the wings would be evident.

Figures 6a and 6b show the output from an array of 81 correlators stored in 250µm thick LiNbO3 displaced 1 cm in front of the Fourier plane. Two different faces in an alternating fashion were used as templates so that the overall correlation pattern could be easily seen. In this experiment the central reference beam angle was 50° and each reference beam was separated by 0.08°. Figures 6c and 6d show the output when the input images are shifted just enough so that their correlation peaks would fall in the area reserved for the neighboring template. Due to the positioning of the hologram in the Fresnel zone the peaks have disappeared as intended. Figures 6e and 6f show the output that results when the holograms are stored in the Fourier plane. In this case, the Bragg-selectivity is not enough to prevent the sidelobes from interfering with neighboring templates and the output of the system is noisy even for well-centered input images. This shows that, using the Fresnel correlator system, more correlators can be stored and viewed unambiguously than would be possible in the Fourier plane. Figure 7 shows cross-sections of auto-correlations for both the Fresnel and Fourier plane holograms. The sidelobes of the Fresnel hologram are clearly suppressed relative to those for the Fourier hologram.

4 Conclusion

It is possible to rely on Bragg-selectivity alone to control the shift invariance of holographic correlator systems. However, simply recording the holograms in the Fresnel zone allows for convenient control without requiring different lenses or thicker recording materials. It also creates symmetric correlation domains which are desirable for certain applications.
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6 References