Doppler signal processing: a new technique

David Casasent and Demetri Psaltis

Carnegie-Mellon University, Department of Electrical Engineering, Pittsburgh, Pennsylvania 15213.

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Realization of an optical Mellin transform has recently been reported and applications of its scale invariance in image processing suggested.\(^1\text{–}^3\) We now report on extensions of these operations to signal processing. Since the effect of the Doppler frequency shift that arises when there is relative motion between the source and receiver is equivalent to a scaling of the time axis from \(t\) to \(bt\),\(^4\) this appears to be a viable technique, the optical implementation will be emphasized because of the parallel processing and high speed advantages of an optical system.

For backscattered radiation in radar, the detected frequency \(\omega_d\) is related to the emitted radiation \(\omega_0\) and the radial velocity \(v_r\) of the source object by

\[
\omega_d = \omega_0 (1 + 2v_r/c),
\]

where \(v_r = \dot{R}_o\) is the rate of change of range \(R_o\) and \(v_r = c\) is the velocity of propagation of the waves in the medium. This Doppler frequency shift is equivalent to a scaling of the time axis from \(t\) to \(bt\), where \(b = 1 - 2R_o/c\).

It has been demonstrated that the magnitudes of the Mellin transforms \(M_1\) and \(M_2\) of two scaled functions \(f_1(x,y)\) and \(f_2(x,y) = f_1(bx,by)\) are identical.\(^1\) This property has been used to correlate scaled input imagery with no loss in the SNR of the correlation peak from the autocorrelation case.\(^2\) Of particular interest in the case of Doppler signal processing is the fact that the correlation peak in Mellin correlation is shifted from its on-axis location by \(K \ln b\), where \(K\) is a constant.\(^2\) Thus, the scale change between \(f_1\) and \(f_2\) (or in this case the Doppler shift and, hence, the velocity of the target) can be determined from the location of the correlation peak. The Mellin transform of \(f(x)\) can be realized by forming the Fourier transform of the scaled function \(f(\exp x)\), where the scaling is obtained by a log module in the deflection system of the input transducer on which \(f(x)\) is recorded.\(^1\)

To demonstrate the use of Mellin transforms in Doppler signal processing, the logarithmically scaled input signal \(f_1\) was recorded on nine lines in the left half of the input plane and the nine logarithmically scaled Doppler shifted versions \(f_2\) of it on the same nine lines but in the right half of the input plane. The transmittance of the input plane is then described by

\[
\sum_n [f_1(x + x_n, y - ny_o) + f_2(x - x_n, y + ny_o)],
\]

where \(n = 9\) rows, the separation between rows is \(y_o\), and the spacing between signals on a given line is \(2x_o\). The one-dimensional Fourier transform of this input distribution was recorded on film, and a second one-dimensional transform of this film pattern obtained. This output pattern contains the correlations \(f_{n1} \times f_{n2}\) at \(x_n' = 2x_o - x_n\). This is a modified version of a joint-transform correlator.\(^5\)

A photograph of the region of the output plane around \(x' = 2x_o, y' = 0\) (displayed on a thresholded monitor) is shown in Fig. 1. The relative location \(x_n'\) of the correlation peak on each line is proportional to the logarithm of the scale change between the signals on line \(n\) in the input and, thus, the Doppler shift between these signals. All correlation peaks are of the same intensity, and only their location now contains information. For convenience, a constant Doppler increment was applied to each of the nine reference signals \(f_{n2}\).

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References